Digitalna tehnika

Logične funkcije in
osnovna digitalna vezja

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Tri osnovne Boolove funkcije nad logičnimi spremenljivkami

disjunkcija
logični ali (+, $\vee$)

Izjavna tabela:

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

Lastnosti disjunkcije:

$$A + 0 = A$$
$$A + 1 = 1$$
$$A + A = A$$
$$A + \overline{A} = 1$$

konjunkcija
logični in ($\cdot$, $\wedge$, & , brez znaka)

Izjavna tabela:

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Lastnosti konjunkcije:

$$A \cdot 0 = 0$$
$$A \cdot 1 = A$$
$$A \cdot A = A$$
$$A \cdot \overline{A} = 0$$

negacija
logični ne (črta nad spremenljivko ali izrazom, \, $\neg$)

Izjavna tabela:

x	y
0	1
1	0

Postulati (aksiomi) in teoremi

	$Postulati\ (aksiomi)$
1	$X + 0 = X$
$1'$	$X \cdot 1 = X$
2	$X + Y = Y + X$
$2'$	$X \cdot Y = Y \cdot X$
3	$X + (Y \cdot Z) = X + Y \cdot Z =$
	$= (X + Y) \cdot (X + Z)$
$3'$	$X \cdot (Y + Z) = X \cdot Y + X \cdot Z$
4	$X + \overline{X} = 1$
$4'$	$X \cdot \overline{X} = 0$

	$Teoremi(izreki)$
1	$X + 1 = 1$
2	$X + X = X$
3	$X \cdot X = X$
4	$\overline{\overline{X}} = X$
5	$X \cdot 0 = 0$
6	$X + XY = X$
7	$X \cdot (X + Y) = X$
8	$(X + \overline{Y}) \cdot Y = XY$
9	$X \cdot \overline{Y} + Y = X + Y$
10	$(X + Y) + \overline{X} = 1$
11	$(\overline{X} \cdot \overline{Y}) \cdot X = 0$
12	$\overline{X + Y} = \overline{X} \cdot \overline{Y}\ (de\ Morgan)$
13	$\overline{X \cdot Y} = \overline{X} + \overline{Y}\ (de\ Morgan)$

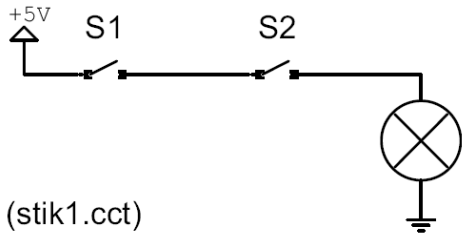
Postulati (aksiomi) in teoremi

Primer:

$$
\begin{aligned}
Y &= (AB(C+D+E)+DBA)\cdot(\overline{A}+\overline{B}+\overline{E}) \\
&= (\underline{AB(C+D+E)}+DBA)\cdot(\overline{A}+\overline{B}+\overline{E}) \\
&= (ABC+\underline{ABD}+ABE+\underline{DBA})\cdot(\overline{A}+\overline{B}+\overline{E}) \\
&= (ABC+ABD+ABE)\cdot(\underline{\overline{A}+\overline{B}+\overline{E}}) \\
&= (ABC+ABD+ABE)\cdot\overline{ABE} \\
&= ABC\overline{ABE}+ABD\overline{ABE}+\underline{ABE\overline{ABE}} \\
&= ABC\overline{ABE}+ABD\overline{ABE} \\
&= AB(C+D)\overline{ABE} \\
&= AB(C+D)(\overline{AB}+\overline{E}) \\
&= \underline{AB(C+D)\overline{AB}}+AB(C+D)\overline{E} \\
&= AB\overline{E}(C+D) = ABC\overline{E}+ABD\overline{E}
\end{aligned}
$$

Izvedba Boolovih funkcij s stikali

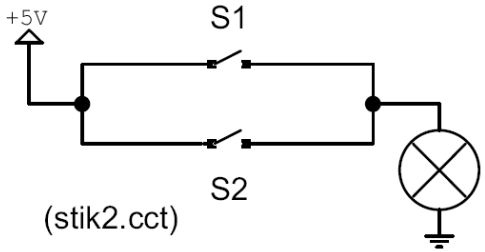
Logični in



(stik1.cct)

$S_1 \cdot S_2$

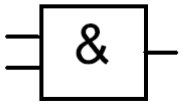
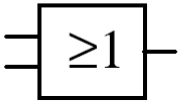
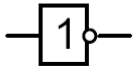
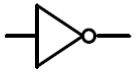
Logični ali



(stik2.cct)

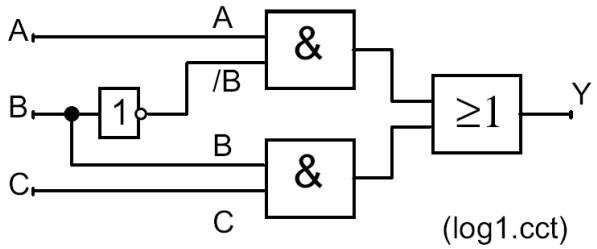
$S_1 + S_2$

Osnovna logična vrata

	Simbol po IEC standardu	Simbol po ANSI standardu
IN (AND)		
ALI (OR)		
NE (NOT, inverter)		

Osnovna logična vrata

Primera:

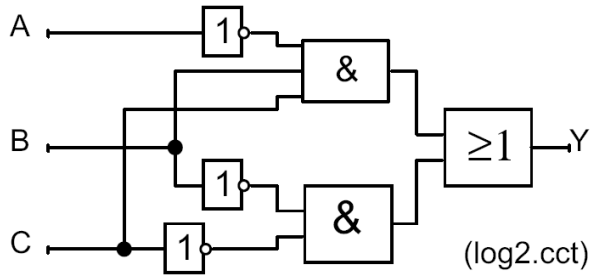


(log1.cct)

$$Y = A \cdot \overline{B} + B \cdot C$$

Izjavna tabela:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



(log2.cct)

$$Y = \overline{A} \cdot B \cdot C + \overline{B} \cdot \overline{C}$$

Izjavna tabela:

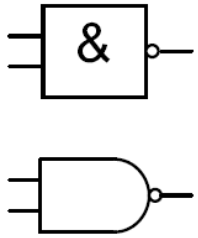
A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Druga logična vrata

NE IN (NAND)

Shefferjeva funkcija

$$y = \overline{x_1 \cdot x_2}$$



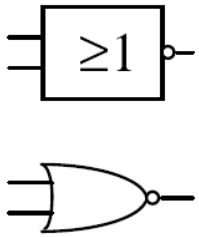
Karakteristična tabela:

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

NE OR (NOR)

Pierceova funkcija

$$y = \overline{x_1 + x_2}$$



Karakteristična tabela:

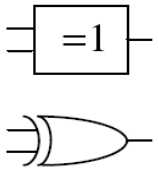
x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

Druga logična vrata

Ekskluzivna ALI

EXOR funkcija
antivalenca

$y = x_1 \oplus x_2$

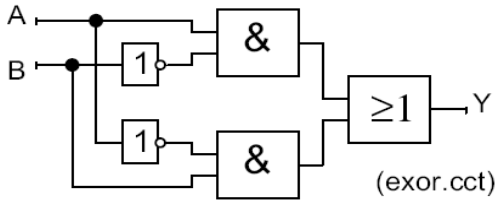


Karakteristična tabela:

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

izvedba z osnovnimi vrati
(iz karakteristične tabele:)

$Y = \overline{A}B + A\overline{B}$



(exor.cct)

Večvhodna EXOR vrata:
Liho število enic na vhodih da enico na izhodu.

Primer uporabe: generator sode paritete

Če je na vhodu liho število enic, bo izhod 1, sicer pa bo 0.

Karakteristična tabela:

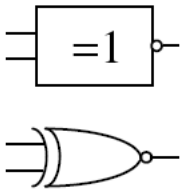
x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Druga logična vrata

Ekskluzivna NE ALI

EX NOR funkcija
ekvivalenca

$y = \overline{x_1 \oplus x_2}$

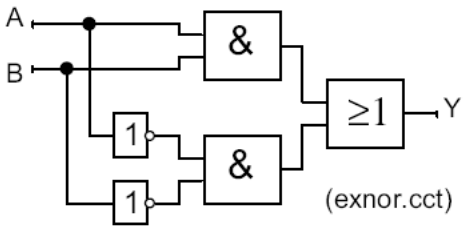


Karakteristična tabela:

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

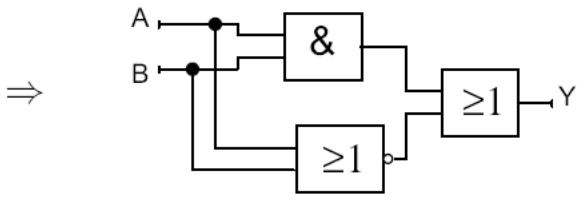
izvedba z osnovnimi vrati
(iz karakteristične tabele:)

$Y = AB + \overline{A}\,\overline{B}$

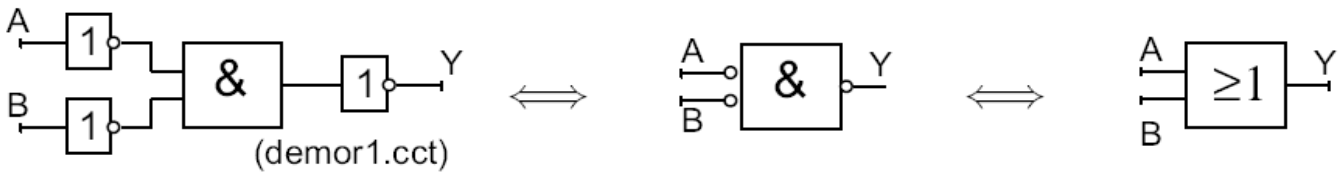


(exnor.cct)

ali po de Morganu:
$Y = AB + \overline{A + B}$

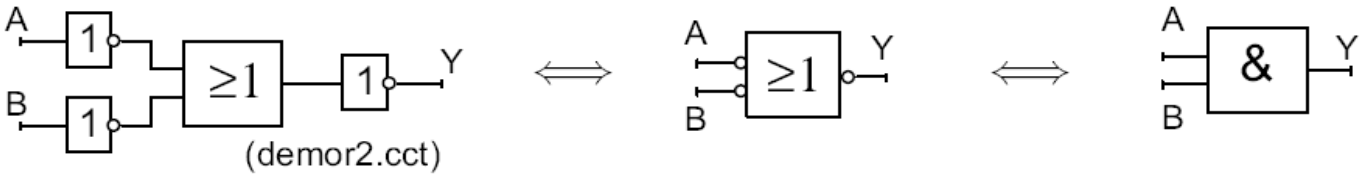


Izvedba vezij z NAND vrati



Po de Morganu ($\overline{A} \cdot \overline{B} = \overline{A + B}$):

$$\overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A + B}} \Rightarrow \overline{\overline{A} \cdot \overline{B}} = A + B$$



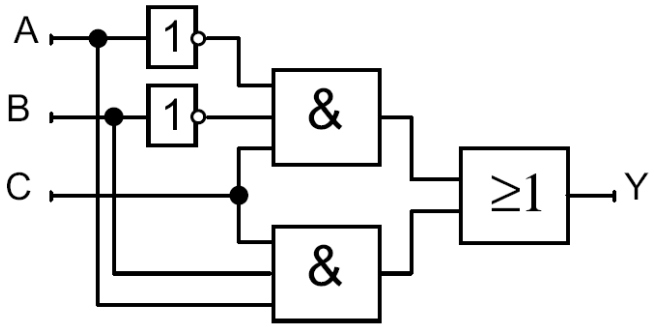
Po de Morganu ($\overline{A} + \overline{B} = \overline{A \cdot B}$):

$$\overline{\overline{A} + \overline{B}} = \overline{\overline{A \cdot B}} \Rightarrow \overline{\overline{A} + \overline{B}} = A \cdot B$$

Kanonične oblike, mintermi, makstermi

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

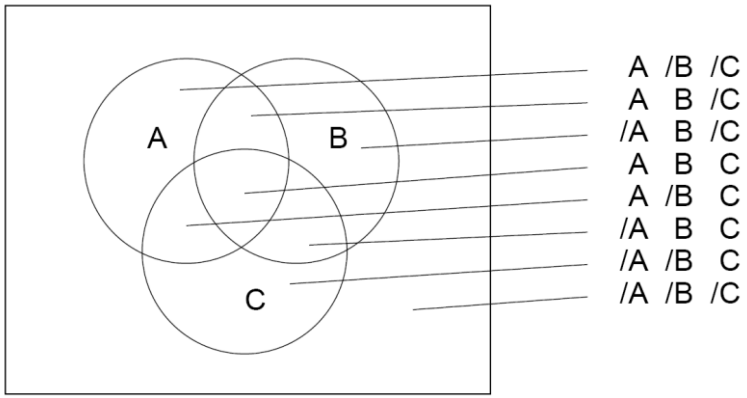
$$Y = \overline{A}\,\overline{B}C + ABC$$



(mintermi.cct)

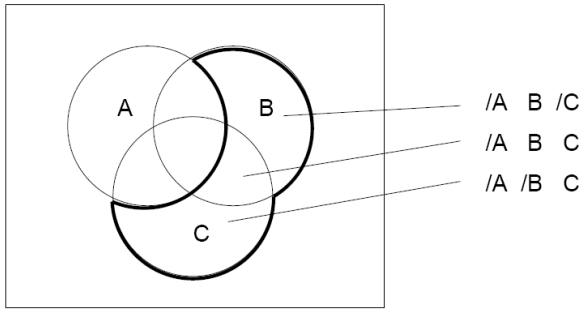
Minimizacija Boolovih funkcij

Vennovi diagrami



Primer: Z uporabo Vennovega diagrama minimiziraj logični izraz $Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

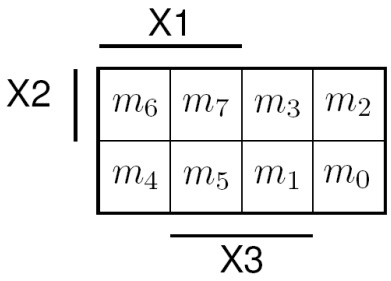


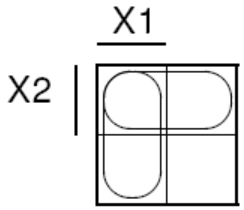
Rezultat: $Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} = \overline{A} \cdot (B + C)$

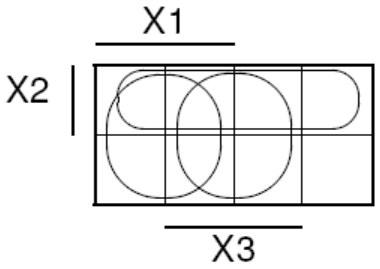
Minimizacija Boolovih funkcij

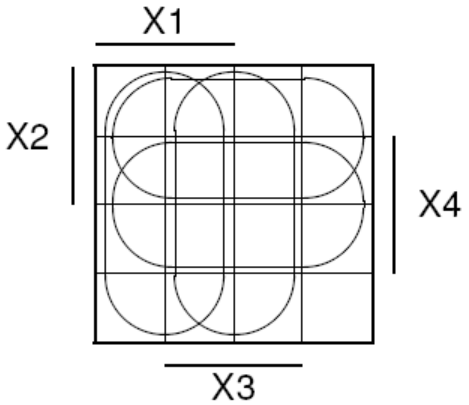
Veitchevi diagrami

X_1	X_2	X_3	Y
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

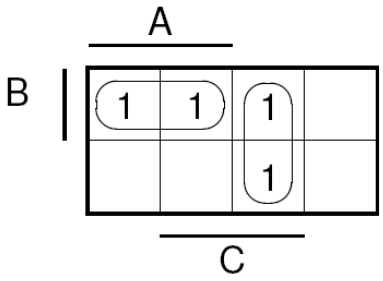




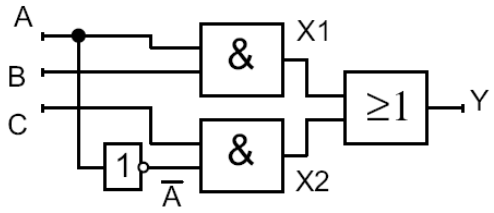


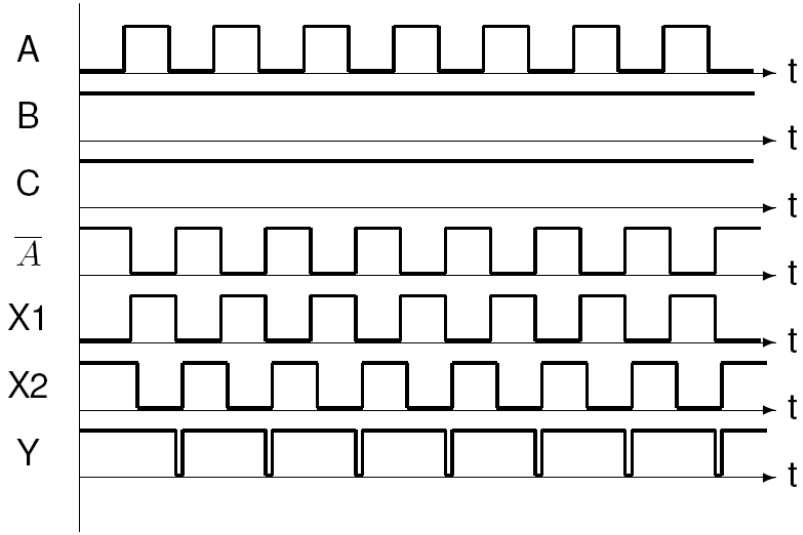


Hazardna stanja pri kombinacijskih vezjih



$$Y = AB + \overline{A}C$$





Izvedba logičnih funkcij s programirljivimi kombinacijskimi vezji

PAL- programirljiva log. polja
PLA- programirljive log. mreže
Pomnilniki - EPROM