

MATEMATIKA V NARAVOSLOVJU

VAJE 8

Vsebina: nedoločeni integral

Lastnosti nedoločenega integrala in nekaj osnovnih integralov

- $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$
- $\int c \cdot f(x)dx = c \int f(x)dx, c \in \mathbb{R}$
- uvedba nove spremenljivke: $\int (f(g(x)) \cdot g'(x)) dx = \int f(u)du; u = g(x), du = g'(x)$
- integracija "per partes": $\int f(x) \cdot g'(x)dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x)dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; \quad n \in \mathbb{R} \setminus \{-1\}$$

$$\int \frac{1}{x} dx = \int \frac{dx}{x} = \int x^{-1} dx = \ln |x| + C$$

$$\int e^x dx = e^x + C; \quad \int e^{kx} dx = \frac{e^{kx}}{k} + C; \quad \int a^x = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C; \quad \int \cos x dx = \sin x + C$$

1. Izračunajte naslednje integrale.

(a) $\int (x^3 - 3x + 2)dx$

(e) $\int x^4 \cdot \sqrt[3]{x} dx$

(b) $\int (4x^4 + x^{-1} - 8)dx$

(f) $\int (x^2 \sqrt[3]{x} - 5 \frac{x}{\sqrt{x^3}} + 2 \frac{2}{\sqrt[4]{x^3}}) dx$

(c) $\int (x^3 - 5)^2 dx$

(g) $\int (1 + e^x + 4 \cdot 2^x) dx$

(d) $\int \frac{x^3 + 3x^2 + 1}{x} dx$

(h) $\int (x - \sin x - 5 \cos x) dx$

2. Z vpeljavo nove spremenljivke izračunajte naslednje integrale

(a) $\int \frac{6x dx}{x^2 - 1}$

(g) $\int \cos x \cdot e^{\sin x} dx$

(b) $\int (x + 3)^3 dx$

(h) $\int \frac{x^4 dx}{(x^5 + 2)^7}$

(c) $\int e^{2x+1} dx$

(i) $\int x \tan(x^2) dx$

(d) $\int x^5 \cdot \sqrt[3]{1 - x^6} dx$

(j) $\int \sin x \tan(\cos x) dx$

(e) $\int \frac{(x-3)dx}{x^2 - 6x + 5}$

(k) $\int \frac{\sin x dx}{e^{\cos x}}$

(f) $\int \cos^2 x \sin x dx$

(l) $\int \frac{(x^2 - 2x)dx}{\sqrt{x^3 - 3x^2 + 1}}$

3. S pomočjo integracije "per partes" izračunajte naslednje integrale.

(a) $\int x \cos x dx$

(e) $\int \ln x dx$

(b) $\int x^2 \sin x dx$

(f) $\int \frac{2x^3 dx}{\cos^2(x^2)}$

(c) $\int x^4 \ln x dx$

(g) $\int e^x \sin x$

(d) $\int \sqrt{x} \ln x dx$

(h) $\int \arcsin^2 x dx$